Low energy strong interactions at high precision

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Collaboration with
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- Introduction:

  Chiral perturbation theory as the low-energy effective theory of the strong interactions

- Simple example: electromagnetic mass difference of kaons and pions

- $\pi\pi$ scattering as a precision test

  - Dispersion relations
  - ChPT at 2 loops
  - Results
  - New experimental results, E865 at Brookhaven, NA48
  - DIRAC experiment
  - Implications to $(g-2)_\mu$

- πK scattering — theory and present status

- $\pi^0 \to \gamma\gamma$

  - Theoretical prediction
  - The PrimEx Experiment at JLab

- Kaon decay form factors:New measurements of KTeV at Fermilab

- Afterword (Baryon sector not discussed here)

Introduction

$$\mathcal{L}_{QCD} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{a\,\mu\nu} + \overline{q} i \gamma^\mu D_\mu q - \overline{q} \mathcal{M} q$$

$\mathcal{M} \to 0$, the left- and right- chiral projections may be rotated independently and the N quark flavors rotated amongst each other.

We have the chiral symmetry given by the group $SU(N)_L \times SU(N)_R$

$SU(N)_L \times SU(N)_R \stackrel{\langle \overline{q} q \rangle}{\to} SU(N)_V$**;** $V = (L + R)$**,** $A = (L - R)$

Corresponding to the $SU(N)_A$ broken symmetry we have $N^2 - 1$ (pseudoscalar) Goldstone bosons.

N=2, Goldstone bosons are $\pi^{\pm}, \pi^0$

N=3, Goldstone bosons are $\pi^{\pm}, \pi^0, K^{\pm}, K^0, \overline{K}^0, \eta$

The Goldstone theorem yields

$$< 0|A_\mu|\pi >= F_\pi p_\mu,$$

and $F_\pi \approx 93$ MeV.

To leading order, $O(p^2)$, $\mathcal{L}^{(2)}_{eff}$ is the non-linear sigma model Lagrangian.

$$\frac{F^2}{2} \nabla_\mu U^T \nabla^\mu U$$

where U is a 4-component real $O(4)$ (note that $O(4) \equiv SU(2) \times SU(2)$) unit vector.

At $O(p^4)$:

$$\mathcal{L}_{eff}^{(4)} = l_1(\nabla^\mu U^T \nabla_\mu U)^2 + l_2(\nabla^\mu U^T \nabla^\nu U)(\nabla_\mu U^T \nabla_\nu U) + \\ l_3(\chi^T U)^2 + l_4(\nabla^\mu \chi^T \nabla_\mu U) + \\ l_5(U^T F^{\mu\nu} F_{\mu\nu} U) + l_6(\nabla^\mu U^T F_{\mu\nu} \nabla^\nu U) + l_7(\tilde{\chi}^T U)^2 + \\ h_1 \chi^T \chi + h_2 \mathrm{tr} F_{\mu\nu} F^{\mu\nu} + h_3 \tilde{\chi}^T \tilde{\chi}$$

where $F_{\mu\nu}$: covariant tensors of external fields and derivatives, and the vectors χ and $\tilde{\chi}$ proportional to external scalar and pseudoscalar fields.

Use this effective lagrangian, loops of the non-linear sigma model and appropriate renormalization, obtain the Green's functions of QCD at this order in the momentum expansion.

At this order, 10 additional coupling constants enter the effective lagrangian.

Alternatively in terms of matrices, e.g., $SU(3)$ chiral perturbation theory. The matrix U then contains the pion and kaon fields: $U = \exp(i\sqrt{2}\Phi/F)$,

$$\Phi = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{bmatrix}$$

Dashen's theorem

- Theorem says that the electromagnetic mass difference of the kaons is equal to that of the pions in the chiral limit.
- Inclusion of electromagnetic effects into the chiral Lagrangian.
- Recent work on estimating these constants using resonance saturation.
- Technically challenging work, accounting for proper renormalization.
- Departure from chiral limit — 'violation' of Dashen's theorem.

$$(\Delta M_K^2)_{EM} - (\Delta M_\pi^2)_{EM} = 1.5(\Delta M_\pi^2)$$

B. Ananthanarayan and B. Moussallam, JHEP 0405:047, 2004

Pion-pion scattering

Pion-pion scattering described by $A(s,t,u)$

The process is

$$\pi^a(p_1) + \pi^b(p_2) \to \pi^c(p_3) + \pi^d(p_4)$$

Isospin conserved by strong interactions; the transition matrix is given by:

$$A(s,t,u)\delta^{ab}\delta^{cd} + A(t,u,s)\delta^{ac}\delta^{bd} + A(u,s,t)\delta^{ad}\delta^{bc}$$

$A(s,t,u) = A(s,u,t)$ (denoted as A_s) generalized Bose statistics $s = (p_1+p_2)^2$, $t = (p_1+p_3)^2$ and $u = (p_1+p_4)^2$, all momenta taken to be incoming.

$\sqrt{s}$ the centre of mass energy ($m_\pi = 1$)

t and u related to the cosine of the centre of mass scattering angle via $\cos\theta = (t-u)/(s-4)$, $s+t+u=4$

Isospin ampliltudes

The s-channel amplitudes for definite iso-spin:

$$T_s^0(s,t,u) = 3A_s + A_t + A_u$$
$$T_s^1(s,t,u) = A_t - A_u$$
$$T_s^2(s,t,u) = A_t + A_u$$

Leading order (Weinberg)

$$A(s,t,u) = \frac{s-1}{32\pi F_\pi^2}$$

At one-loop order, loops generate correct analytic structure

At $O(p^4)$ 4 scale free coupling constants $\bar{l}_{1,2,3,4}$ enter the $\pi\pi$ scattering amplitude.

In our normalization, the partial wave decomposition reads

$$T^I(s,t) = 32\,\pi \sum_\ell (2\ell+1)\, P_\ell\left(1+\frac{2t}{s-4}\right) t_\ell^I(s)$$

$$t_\ell^I(s) = \frac{1}{2i\sigma(s)}\left\{\eta_\ell^I(s)\, e^{2i\delta_\ell^I(s)} - 1\right\}$$

$$\sigma(s) = \sqrt{1-\frac{4}{s}}\,.$$

δ_l^I: phase shifts, η_l^I: elasticity parameters.

The threshold parameters are the coefficients of the expansion

$$\mathbf{Re}\, t_\ell^I(s) = q^{2\ell}\left\{a_\ell^I + q^2\, b_\ell^I + q^4\, c_\ell^I + \ldots\right\}$$

with $s = 4(1+q^2)$.

a_l^I: scattering lengths, b_l^I: effective ranges

Weinberg: LO prediction for $a_0^0 = 7/(32\pi F_\pi^2) \simeq 0.16$.

At $O(p^4)$ infrared singularities modify prediction substantially, expressed in terms of the four $\bar{l}$'s, in addition to F_π.

Estimates from disparate sources such as D- wave scattering lengths [alternatively from $\pi\pi$ phase information directly], $SU(3)$ mass relations and F_K/F_π, correction of about 25% to the LO prediction,

Gasser & Leutwyler $a_0^0 = 0.20 \pm 0.01$

Dispersion relations for the t-channel isospin amplitudes

Froissart bound $\rightarrow$ 2 subtractions

$$T_t^I(s,t,u) = \mu_I(t) + \nu_I(t)(s-u) + \frac{1}{\pi}\int_4^\infty \frac{ds'}{s'^2}\left(\frac{s^2}{s'-s} + (-1)^I \frac{u^2}{s'-u}\right)\sum_{I'} C_{st}^{II'} A_s^{I'}(s',t)$$

$\mu_I(t)$, $\nu_I(t)$ are unknown t-dependent

Subtraction constants ($\mu_1 = \nu_0 = \nu_2 = 0$), $A_s^I(s',t)$ absorptive part of the s-channel amplitude. C_{st} crossing matrix, the entries of which are

$$C_{st}(c,d) = (-1)^{(c+d)}(2c+1)\begin{Bmatrix} 1 & 1 & d \\ 1 & 1 & c \end{Bmatrix}$$

Roy representation

$$\begin{aligned} T_s^I(s,t) &= \sum_{I'} \tfrac{1}{4}(s\,1^{II'} + t\,C_{st}^{II'} + u\,C_{su}^{II'})\,T_s^{I'}(4,0) \\ &+ \int_4^\infty ds'\,g_2^{II'}(s,t,s')\,A_s^{I'}(s',0) + \int_4^\infty ds'\,g_3^{II'}(s,t,s')\,A_s^{I'}(s',t)\,. \end{aligned}$$

For our purposes, it is convenient to write the kernels in the form

$$\begin{aligned} g_2(s,t,s') &= -\frac{t}{\pi\,s'\,(s'-4)}\,(u\,C_{st} + s\,C_{st}\,C_{tu})\left(\frac{1}{s'-t} + \frac{C_{su}}{s'-4+t}\right) \\ g_3(s,t,s') &= -\frac{s\,u}{\pi\,s'(s'-4+t)}\left(\frac{1}{s'-s} + \frac{C_{su}}{s'-u}\right)\,. \end{aligned}$$

Furthermore, $T_s(4,0) = 32\pi(a_0^0, 0, a_0^2)$.

Roy's representation for the partial wave amplitudes t_l^I of elastic $\pi\pi$ scattering reads

$$t_\ell^I(s) = k_\ell^I(s) + \sum_{I'=0}^{2} \sum_{\ell'=0}^{\infty} \int_{4M_\pi^2}^{\infty} ds' \, K_{\ell\ell'}^{II'}(s,s') \, \mathbf{Im}\, t_{\ell'}^{I'}(s') \,,$$

where I and ℓ denote isospin and angular momentum, respectively and $k_\ell^I(s)$ is the partial wave projection of the subtraction term. It shows up only in the S- and P-waves,

$$k_\ell^I(s) = a_0^I \, \delta_\ell^0 + \frac{s - 4M_\pi^2}{4M_\pi^2} (2a_0^0 - 5a_0^2) \left(\frac{1}{3} \, \delta_0^I \, \delta_\ell^0 + \frac{1}{18} \, \delta_1^I \, \delta_\ell^1 - \frac{1}{6} \, \delta_2^I \, \delta_\ell^0 \right) .$$

The kernels $K_{\ell\ell'}^{II'}(s,s')$ are explicitly known functions

Contain a diagonal, singular Cauchy kernel that generates the right hand cut in the partial wave amplitudes, and logarithmically singular piece that accounts for the left hand cut.

The validity of these equations has rigorously been established on the interval $-4M_\pi^2 < s < 60M_\pi^2$.

The comparison of the axiomatic and chiral representations of the scattering amplitude yields sum rules for the low-energy constants.

$$\begin{aligned}
\alpha_0^I &= a_0^I - \frac{4}{\pi}\int_4^\infty \frac{dx}{x(x-4)}\mathrm{Im} f_0^I(x) + \frac{4}{\pi}\int_4^\infty \frac{dx}{x^2}\mathrm{Im} f_0^I(x) \qquad I = 0,2 \\
\gamma_0^I &= \frac{1}{\pi}\int_4^\infty \frac{dx}{x^3}\mathrm{Im} f_0^I(x) \qquad I = 0,2 \\
\beta_1^1 &= \frac{3}{\pi}\int_4^\infty \frac{dx}{x^2(x-4)}\mathrm{Im} f_1^1(x) \\
\alpha_0^1 &= \gamma_0^1 = \beta_1^0 = \beta_1^2 = 0.
\end{aligned}$$

In particular we have for $\bar{l}_1$ and $\bar{l}_2$:

$$\begin{aligned}
\bar{l}_1 &= 24\pi^2 {F_\pi}^4\left(\frac{41}{960\pi^2 {F_\pi}^4} - \frac{64\pi}{3}(\gamma_0^2 - \gamma_0^0 + 3\beta_1^1)\right), \\
\bar{l}_2 &= 24\pi^2 {F_\pi}^4\left(\frac{29}{480\pi^2 {F_\pi}^4} + 32\pi(\beta_1^1 + \gamma_0^2)\right).
\end{aligned}$$

For the numerical values we find for $\bar{l}_1$ and $\bar{l}_2$:

$$\begin{aligned}
\bar{l}_1 &= -1.7 \pm 0.15 \\
\bar{l}_2 &= 5.0.
\end{aligned}$$

The strategy: solve these equations numerically using accurately known phase-shift information from medium and high energy region to determine the low-energy parameters.

Sources are $\pi N \to \pi\pi N$, $e^+e^- \to \pi^+\pi-$.

High energy information comes from 'theory', viz., Veneziano model, Pomeron, etc..

Recent comprehensive analysis is reported in

B. Ananthanarayan, G. Colangelo, J. Gasser and H. Leutwyler, Physics Reports 353 (2001) 207.

Self-consistent solutions in the near threshold region yield a_0^0 and a_0^2.

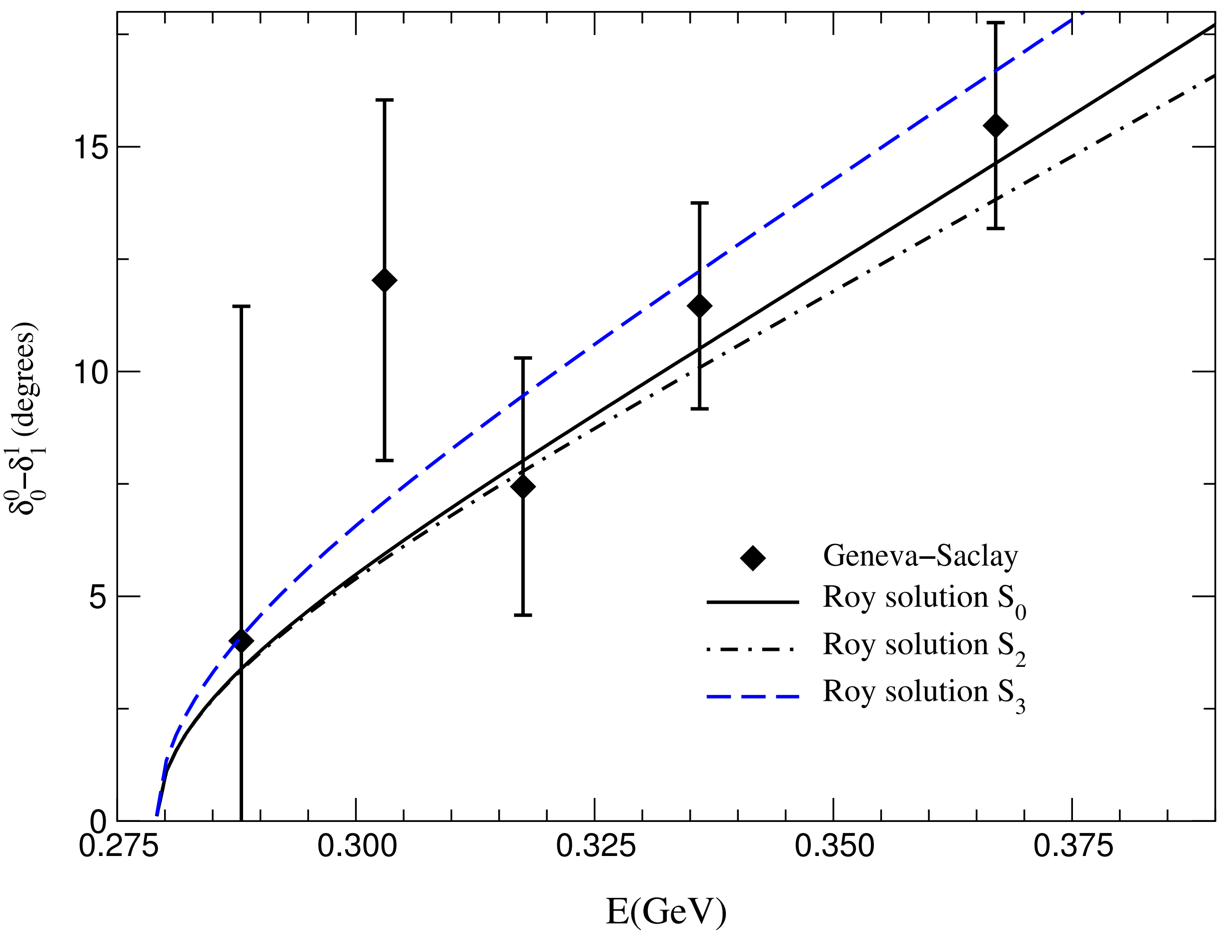
G. Colangelo, J. Gasser, H. Leutwyler, Physics Letters B488 (2000) 261; Physical Review Letters 86 (2001) 5008; Nuclear Physics B603 (2001) 125

Precise determination is also a test of the standard picture of chiral symmetry breaking. "Generalized chiral perturbation theory" of Stern and collaborators. Now ruled out.

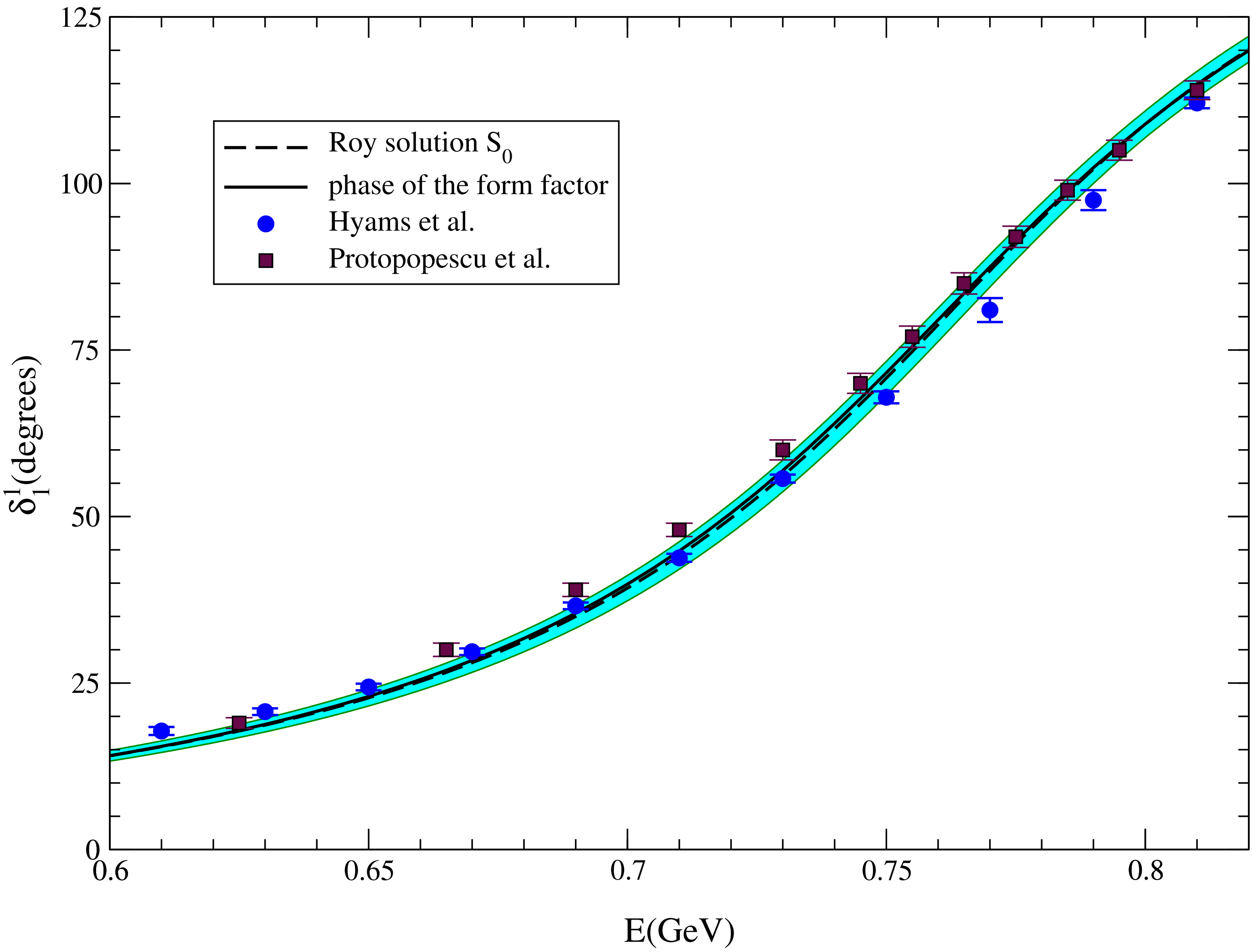
The main process considered here is the rare decay K_{l4} where the final state $\pi\pi$ interaction yields the window to the phase shifts. Comes from the Pais-Treiman method.

Recent measurements at E 865, Brookhaven National Laboratory

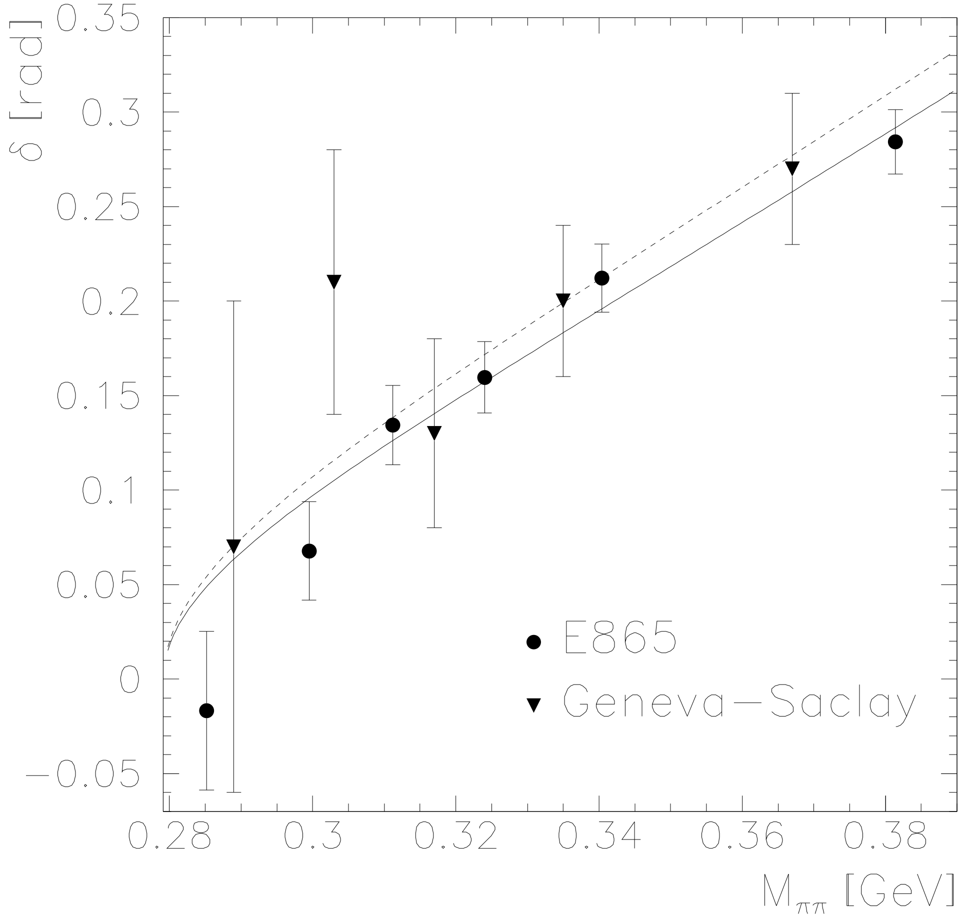
Experimental determination of the phase shift difference $\delta_0^0 - \delta_1^1$ From ACGL



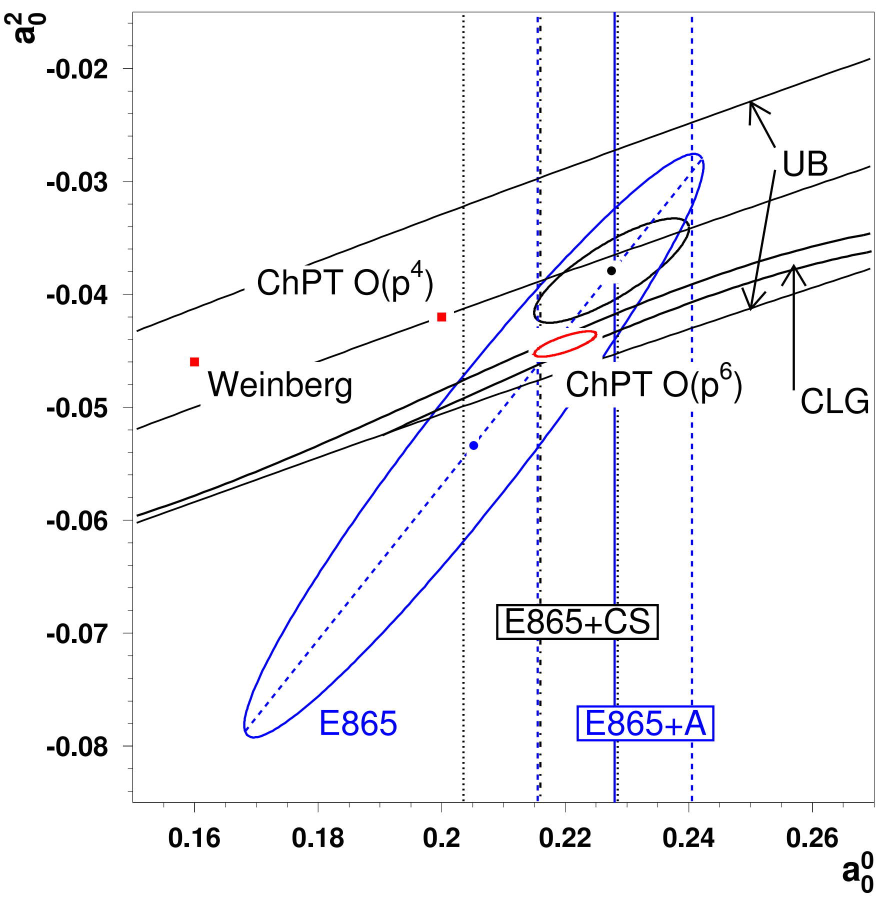
The ρ shape from Roy equation fits and from electromagnetic form factor of the pion, Gounaris-Sakurai fit.



New determination of the phase-shift difference from the E 865 collaboration from 400, 000 reconstructed events. **S. Pislak, et al., Physical Review D 67 (2003) 072004; Physical Review Letters, 87 (2001) 221801**



E 865's report on the scattering lengths



PIONIUM ATOMS

Theory — 'Deser's theorem'

(S.Deser, M. L. Goldberger, W. Baumann and W. Thirring, Physical Review 96 (1954) 774.)

Modern relativistic bound state theory — due to the Bern group.

(See, e.g., J. Gasser, V. E. Lyubovitskij, A. Rusetsky and A. Gall, Physical Review D 64 (2001) 016008)

$$\tau = 2.9 \pm 0.1 \text{ fs}$$

The DI(meson)R(elativistic)A(tom)C(omplex) experiment at CERN Experiment: PS212

- Attempts to measure the lifetime of the pionium ($\pi^+\pi^-$) atom in its groundstate, of about 3 fs at the 10% level.
- The bound state arises due to the electromagnetic interaction, and then through the strong interaction decays into $\pi^0\pi^0$.
- The lifetime is related to the difference

$$a_0^0 - a_0^2$$

- The measurement is in medium.
- Ni Run already completed. $n_A \sim 6600$
- Preliminary lifetime measurement is $\tau = 2.85^{+0.48}_{-0.41}$ fs

Accurate lifetime measurement is expected at the 5% level.

Implications to $(g-2)_\mu$:

The leading uncertainties to the standard model value now comes from hadronic uncertainties.

Celebrated example is the hadronic light by light scattering (Knecht and Nyffeler)

Uncertainties in the pion form factor due to inconsistent (?) data sets of CLEO and Novosibirsk. Also the ALEPH data from τ decay.

Does not allow a test of the standard model.

What about πK atom?

Proposal has been sent to the CERN scientific council.

πK scattering

We consider the process

$$\pi^{I_1}(p_1) + K^{J_1}(q_1) \to \pi^{I_2}(p_2) + K^{J_2}(q_2),$$

with the four-momenta p_i, q_i and the isospin I_i and J_i of the pions and the kaons, respectively. The Mandelstam variables are defined as ($\Sigma \equiv M^2 + m^2$)

$$s = (p_1 + q_1)^2,\ t = (q_1 - q_2)^2,\ u = (q_1 - p_2)^2,$$

with

$$s + t + u = 2\Sigma,$$

where M and m are the pion and the kaon mass, respectively. In the s-channel the center of mass scattering angle Θ_s and momentum q_s are given by ($\Delta \equiv M^2 - m^2$)

$$z_s \equiv \cos\Theta_s = 1 + \frac{t}{2q_s^2} = \frac{t - u + \frac{\Delta^2}{s}}{4q_s^2},$$

$$q_s^2 = \frac{(s - (m - M)^2)(s - (m + M)^2)}{4s},$$

and the partial wave decomposition is defined by

$$T^{I_s}(s,t,u) = 16\pi \sum (2l+1) f_l^{I_s}(s) P_l(z_s).$$

The partial waves may then be parametrized by the phase shifts δ_l^I and the elasticities η_l^I,

$$f_l^I(s) = \frac{\sqrt{s}}{2q_s}\frac{1}{2i}\left\{\eta_l^I(s) e^{2i\delta_l^I(s)} - 1\right\},$$

and have the threshold expansion

$$\mathrm{Re}\, f_l^I(s) = \frac{\sqrt{s}}{2} q^{2l}\left\{a_l^I + b_l^I q^2 + O(q^4)\right\}.$$

In the t-channel, the center of mass momenta of the pion and the kaon are q_t and p_t, respectively, and the centre of mass scattering angle Θ_t is given by

$$
\begin{aligned}
z_t &\equiv \cos\Theta_t = \frac{s + p_t^2 + q_t^2}{2 q_t p_t} = \frac{s-u}{4 p_t q_t},\\
p_t &= \sqrt{\frac{t-4m^2}{4}},\; q_t = \sqrt{\frac{t-4M^2}{4}}.
\end{aligned}
$$

The partial waves are defined by

$$
T^{I_t}(s,t,u) = 16\pi\sqrt{2}\sum (2l+1) f_l^{I_t}(t) P_l(z_t).
$$

Once one of the isospin amplitudes is known the other and combinations of these are fixed by crossing symmetry:

$$
\begin{aligned}
T^{1/2}(s,t,u) &= \frac{3}{2}T^{3/2}(u,t,s) - \frac{1}{2}T^{3/2}(s,t,u),\\
T^{+}(s,t,u) &\equiv \frac{1}{3}T^{1/2}(s,t,u) + \frac{2}{3}T^{3/2}(s,t,u), = \frac{1}{\sqrt{6}}T^{I_t=0}(s,t,u),\\
T^{-}(s,t,u) &\equiv \frac{1}{3}T^{1/2}(s,t,u) - \frac{1}{3}T^{3/2}(s,t,u) = \frac{1}{2}T^{I_t=1}(s,t,u)
\end{aligned}
$$

It may be seen from the above that $T^{+}(s,t,u)$ is even under the interchange of s and u, whereas $T^{-}(s,t,u)$ is odd. The fixed-t dispersion relation for T^{+} is given by

$$
T^{+}(s,t,u) = 8\pi(m+M)a_0^{+} +
$$
$$
\tfrac{1}{\pi}\int_{(m+M)^2}^{\infty} \tfrac{ds'}{s'^2}\left[\tfrac{s^2}{s'-s} + \tfrac{u^2}{s'-u}\right] A_s^{+}(s',t) + S^{+} + L^{+}(t) + U^{+}(t).
$$

The expressions for $S^{+}, L^{+}(t)$, and $U^{+}(t)$ are known functions. These are obtained by combining fixed-t and hyperbolic dispersion relations. An analogous dispersion relations can be written for

$$
\frac{T^{-}(s,t,u)}{(s-u)}
$$

All the results discussed above are from

B. Ananthanarayan and P. Büttiker, European Physical Journal C 19 (2001) 517; B. A., P. B. and B. Moussallam, ibid. C 22 (2001) 133

The aim is to use all the known information from phase shift analysis and chiral inputs to render the process predictive. Becomes a sensitive test of the full $SU(3)$ chiral perturbation theory, viz., of the expansion in the s-quark mass.

Amplitude has been computed by Bernard, Kaiser and Meißner

System of sum rules for various low-energy constants. Unprecedented determination has been carried out.

COMPASS experiment at CERN.

$\pi^0 \to \gamma\gamma$

- **Adler, Bell and Jackiw: chiral anomalies**

$$\partial^\mu A^3_\mu = \frac{\alpha}{4\pi} F\tilde{F} \Rightarrow \kappa = \frac{\alpha}{4\pi F_\pi}$$

  Predicts

$$\Gamma_{\pi^0\to\gamma\gamma} = 7.73 \quad \text{eV}$$

  within one standard deviation of current world-average experimental value.

- **QCD: anomaly understood at the quark level; same result.**

- **ChPT allows to determine corrections due to chiral-symmetry breaking by quark masses**

  In $SU(2)$ NLO corrections, but LO in $SU(3)$: they are $+2\%$

  NLO corrections: they are -0.3%

  B. Ananthanarayan and B. Moussallam, JHEP 0205:052, 2002

  J. Goity, A. M. Bernstein, B. Holstein, Physical Review D 66 (2002) 076014

Experimental Status **World average value:** $\Gamma_{\pi^0\to\gamma\gamma} = 7.74 \pm 0.55$ **eV. 7% error bar. Today: PRIMEX at JLab: Primakoff with aimed error of** $\simeq 1.5\%$

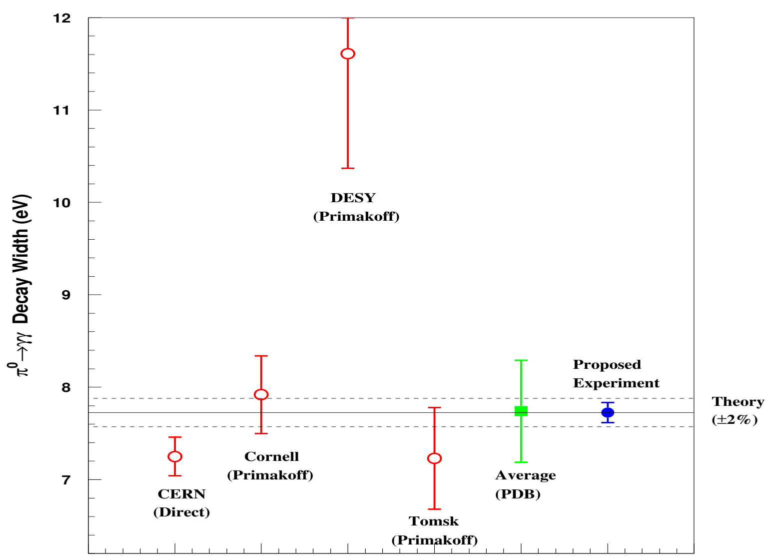


Figure 1: $\pi^o \to \gamma\gamma$ decay width in eV. The horizontal line is the prediction of the axial anomaly(Eq. 2)[1, 2] with an estimated 2% error[7]. The experimental results with errors are for : 1) the direct method[9]; 2,3,4) the Primakoff method [11, 12, 13]; 5) Particle Data Book Average [3]; 6) the expected error for our future experiment, arbitrarily plotted to agree with the predicted value.

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From a talk by A. Gasparian

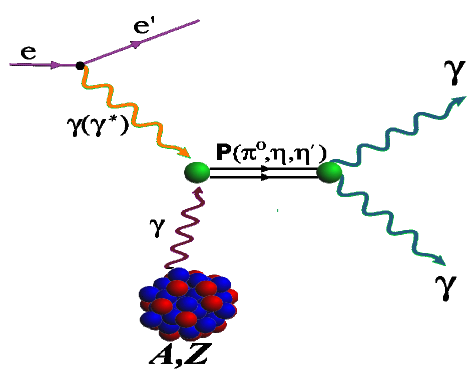
PAC23, January 18, 2003

The Experimental Program

We propose to measure:

- **Two-Photon Decay Widths:** $\Gamma(\pi^0 \to \gamma\gamma)$, $\Gamma(\eta \to \gamma\gamma)$, $\Gamma(\eta' \to \gamma\gamma)$
- **Transition Form Factor** $F_{\gamma\gamma^* P}$ **of** π^0, η, η' **at low** Q^2 **(0.001–0.5 GeV^2)**

via the *Primakoff effect.*



Form Factor Measurements by KTeV (E832) Experiment

- **800 GeV/c proton beam strikes a BeO target**
- $K_L \to \pi^{\pm} l^{\mp} \nu$ **decay modes are studied**
- **1.9 million events for e, 1.5 million events for** μ
- **Necessary for extraction of Cabibbo-Kobayashi-Maskawa matrix element** $|V_{us}|$

T. Alexopoulos et al., hep-ph/0406003

Afterword

- Chiral perturbation theory is the low-energy effective theory of the standard model.

  Worked out to two-loop order for many processes.

  Provided impetus of many innovative new experiments.

- Dashen's theorem for electromagnetic mass difference.

- Remarkable synthesis of dispersion relation phenomenology and effective lagrangian theory.

- $\pi\pi$ scattering now worked out to an unprecedented level of accuracy.

  Remarkable new experiments (E865, DIRAC)

- πK scattering is the next setting for such a state of affairs.

- $\pi^0 \to \gamma\gamma$ is a sensitive laboratory and will be tested at PrimEx.

- Form factor measurements at KTeV