From Eden to Invasion: Domain Growth in Field-driven Random-Field Ising Model

Ratnadeep Roy

The Institute of Mathematical Sciences,Chennai
India

(TPSC, SNBNCBS Kolkata)

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Collaborator: Purusattam Ray

- Hamiltonian for RFIM :

$$H = -J \sum_{<ij>} s_i s_j - \sum_i h_i s_i - h_a \sum_i s_i$$

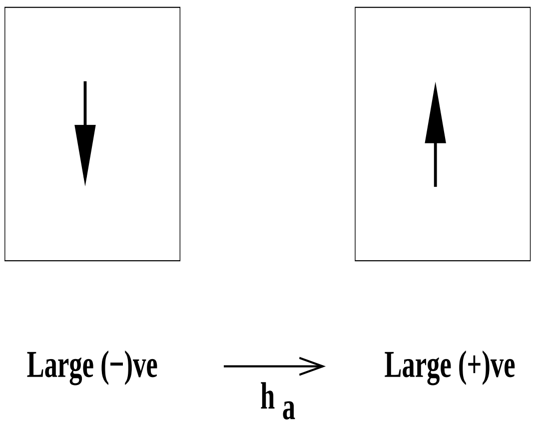
h_i is the quenched random-field on site i with $p(h_i)$

h_a is the applied external field

- RFIM driven by an external field is used to study:

  * Driven disordered systems
  * Hysteresis,Barkhausen noise
  * Effect of disorder on first order transitions in general

Methodology:

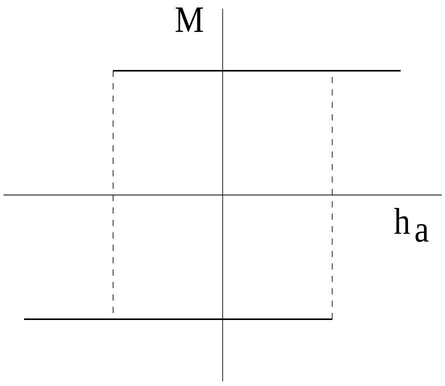


Spin Flip Rule: T=0 Glauber dynamics

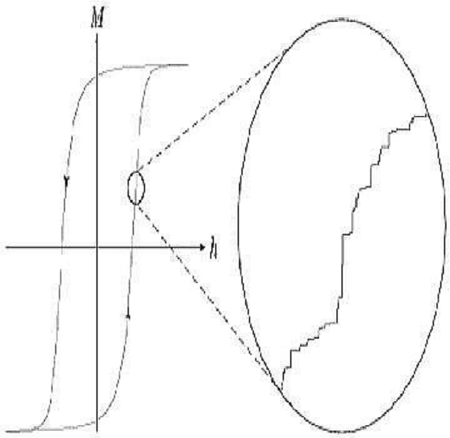
- Spin direction is given by the sign of local field at that site

$$s_i = \mathrm{sign}(l_i) = \mathrm{sign}(J \sum_{j=1}^{z} s_j + h_i + h_a)$$

h_a is incremented only when all flippable spins have been flipped for a certain h_a.



M vs. h_a for a pure-system($h_i = 0$)



M vs. h_a for a disordered system

- Sharp first order transition is smeared out
- change in M occurs in small jumps/ avalanches

Mean-Field Analysis :

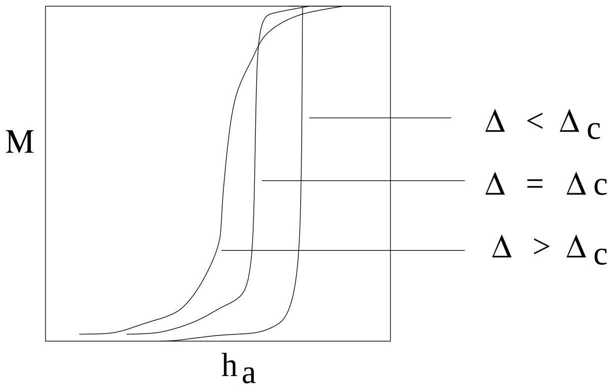
- Every spin is coupled to all other spins
- The effective field acting on a site

  $JM + h_i + h_a$, $M = \sum_i s_i/N$

- Spins with $h_i < -JM - h_a$ will point down, rest points up

$$M(h_a) = 1 - 2\int_{-\infty}^{-JM(h_a)-h_a} p(h_i)dh_i$$

$M(h_a)$ as a function of h_a can be found self-consistently.



Typical $M - h_a$ curve for $p(h_i)$ - Gaussian with width Δ

- Avalanche size distribution $D(s, h_a)$ at critical values Δ_c and $h_a = h_c$

$$D(s, h_c) \sim s^{-3/2}$$

- At Δ_c, cumulative avalanche size distribution $D(s)$ upto h_c

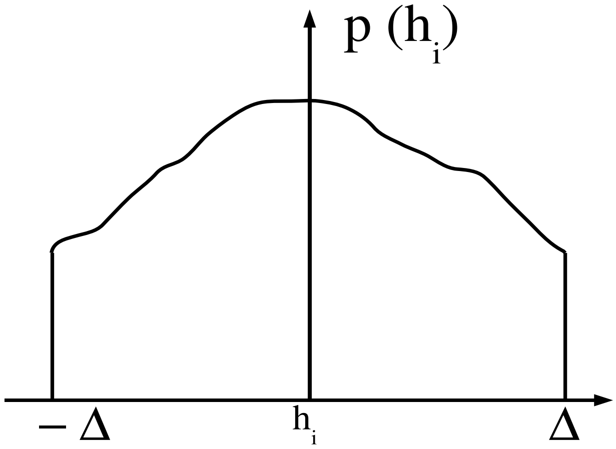
$$D(s) = \int_{-\infty}^{h_c} D(s, h_a) dh_a$$

$$D(s) \sim s^{-5/2}$$

Simulations with $p(h_i)$ gaussian (unbounded) in 3d support mean-field results.

Our Study :

- To check the universality of the mean-field behavior for bounded distribution and at low dimensions



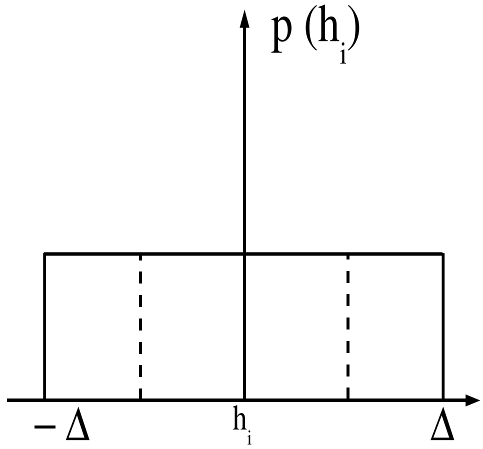
The probability of flipping of a spin at h_a having n flipped nearest neghbors is

$$P_n(h_i) = \int_{(z-2n)J-h_a}^{\Delta} p(h_i) dh_i$$

z = co-ordination number

$h_a = zJ - \Delta$

For a uniform distribution of h_i:

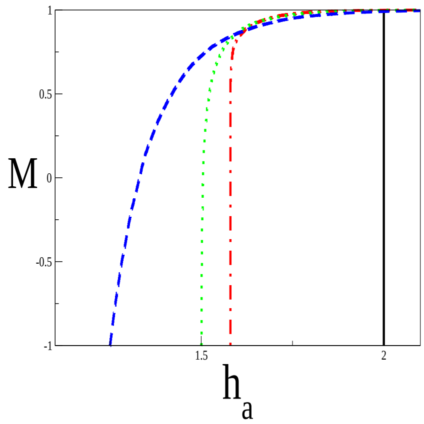


$$P_n(h_i) = n\frac{J}{\Delta}$$

- Rule for growth of Eden clusters
- Flipping probability is proportional to the number of nn flipped spins

Our Simulation:

- **2d**: square, honeycomb, triangular (500^2 to 7000^2)
  3d: simple cubic lattices (50^3 to 200^3)
- Results averaged over 100 different random-field configurations



Features of magnetization curves:

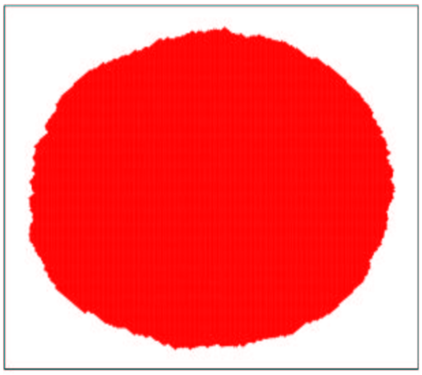
i) For low Δ, M jumps from -1 to +1 at some value of h_a (black line)

ii) For intermediate Δ, M jumps from -1 to a fixed value $M^{'} < M$ at some h_a - value and then increases in small jumps with h_a

iii) At critical Δ_c , magnetization jumps occur in all sizes

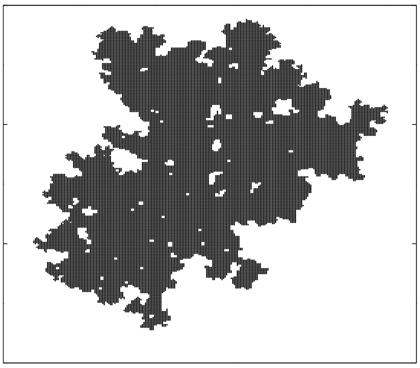
iv) For $\Delta_d > \Delta_c$, M changes in small jumps with increase of h_a

Domain growth corresponding to the various magnetization curves:



a

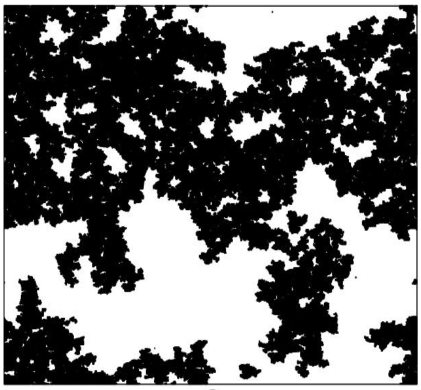
i) Fig.(a) - a snapshot of the flipped spin domain for low Δ. M goes from -1 to +1. Domain is compact.



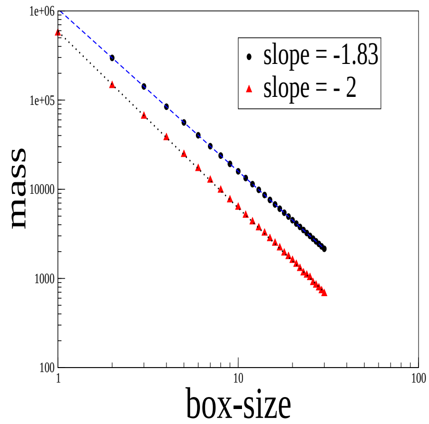
b

ii) Fig.(b) - domain of flipped spins for intermediate Δ.

* Domain is interspersed with clusters of unflipped spins, fractal dimension of the cluster $d_f = 2$
* Unflipped spins responsible for M going to M'
* Domain is system spanning

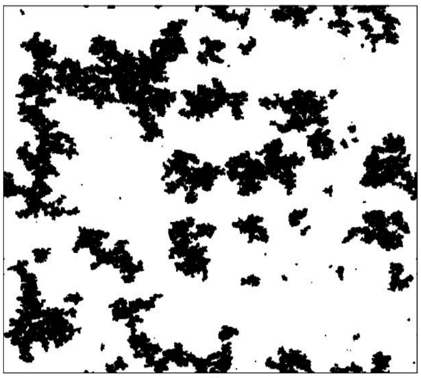


c



iii) Fig.(c) - Flipped spin domain at Δ_c when it has just spanned the system on few increments of h_a.

- Fractal dimension of the cluster is same as that for a 2d Invasion Percolation cluster with traps (black circles).
- Integrated avalanche size distribution has a power-law form with exponent $-3/4$



d

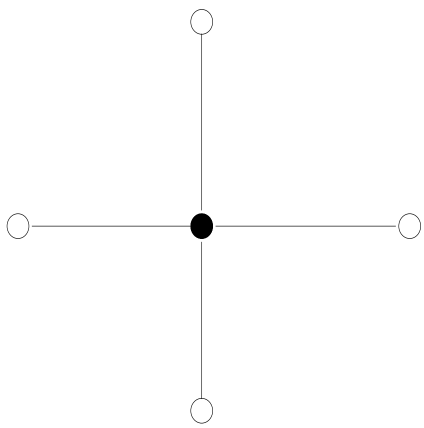
iv) Fig.(d) - a snapshot of the flipped spin domains at large Δ after a few increments of h_a

- Clusters of small domains are scattered all over the lattice

  * shows that spin flip can occur anywhere on the lattice in small avalanches with increment of h_a

Similar results are obtained in 3d.
At Δ_c, the fractal dimension d_f of the flipped spin region is 2.54 (same as that of site percolation cluster in 3d)

Growth model studied:



$$P_1(=P) : P_2(=2P) : P_3(=3P)$$

We find at P_c, $d_f = 1.83$

$1/P_c$ corresponds to Δ_c of RFIM

Conclusion:

i) $M - h_a$ curve depends crucially on $p(h_i)$ specially at low dimensions

ii) For uniform bounded distribution, the critical point corresponds to Invasion percolation

iii) For uniform distribution, RFIM provides an interesting growth model where one can go from compact Eden growth to Invasion Percolation by changing the parameter Δ